POWER LOSS IN ONE DIMENSIONAL CARBON NANO TUBES AND HEAT EVOLVED IN CARBON NANO TUBES VARIES AS THE SQUARE OF THE LENGTH

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ABSTRACT

The field of carbon nano tubes (CNT) is an active and promising area of research theoretically as well as experimentally In fact, the band structure of CNT determines its conductivity and in carbon nano tubes the structural pattern affects the conductivity type i.e the conductivity of CNT depends upon how a graphene sheet is rolled up. So, CNT can be made to behave as conductor as well as semiconductor [. It is well established fact that the arm chair wrapping have been produced and exhibit metallic behavior with an intrinsic resistivity which increases approximately linearly with temperature. In this paper the quantized value of the resistivity has been used with special reference to one dimensional CNT with potential well of infinite depth, in an attempt to review the linear temperature dependence of quantum state (n) in order to calculate the power loss in CNT due to flow of current. Finally, three striking results are obtained. (i) Heat lost in the tube is dependent on temperature (ii) It is also achieved that lost of heat varies directly as the square of the length of the tube.(iii) Finally, it is shown that negative temperature gradient of power is equal to some constant times the temperature co-efficient of resistance.

KEYWORDS: potential well, Electrical conductivity, temperature co-efficient of resistance, quantum state, power loss, relaxation time

I. INTRODUCTION

It has been already established {see for instance [1]} when CNT is extremely narrow (thin) for a very thin conductor $A^{\frac{1}{2}} << L$ where A is the area of cross section and L is the length of the potential well, the total energy of a single electron in side a one dimensional well of infinite depth is given by,

$$E_n = \frac{n^2\pi^2h^2}{2mL^2} ....(i)$$ 'm' is the mass of the electron.

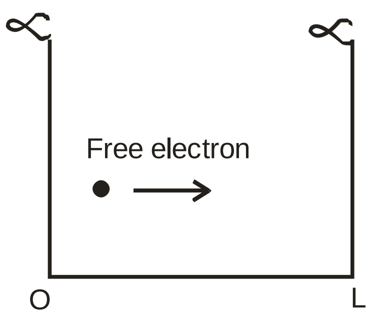


Figure-01

For one electron system, electron spatial density

$$N_n = \frac{1}{AL} ....(ii)$$

Many experiments show that CNT are ballistic conductor [2], and therefore velocity of the electron in the *n*th quantum state $v_n = \frac{nh}{2mL} ....(iii)$

Introducing transit time of the electron in nano tube

$$\tau = \int_O^L \frac{dx}{v_n} = \frac{L}{v_n}$$

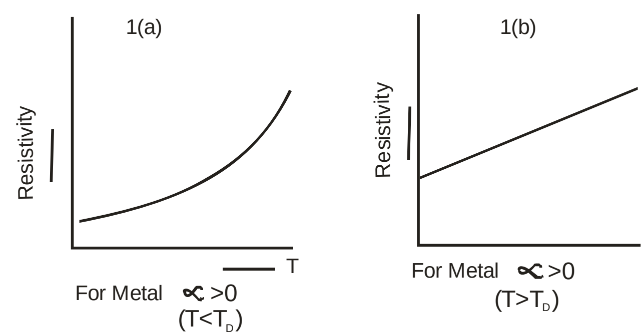
$$\tau_n = 2m\frac{L^2}{nh} ....(iv)$$

With the help of (i), (ii), (iii) and (iv) we could readily establish the relation for conductivity for CNT.

$$\sigma_n = \frac{2Le^2}{Anh} \quad or$$

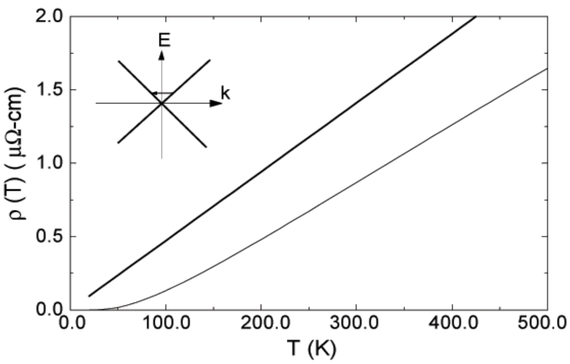
$$\rho_n = \frac{Anh}{2Le^2} ....(v)$$

For pure conductor the resistance increases linearly with temperature in the temperature range around and above the room temperature i.e $\rho \propto T$, see for instance pg no. 184 [7]. It may be pointed out that at very low temperature the resistivity of the material increases as the higher power of temperature, for metallic conductor at very low temperature $\rho \infty T^5$, see for instance pg no. 179[8].



1. Thus, the variation of the resistivity with temperature is also a indispensible part of CNT too. Which is evident from theoretical as well as experimental research [4] Apart from this in one dimensional theory of scattering of electrons by twistons which predicts the intrinsic resistivity varies directly with temperature [4], Moreover, calculated temperature dependence resistivity duet to twiston scattering including both one and three dimensional cases, we have been able to observe that $\rho \propto T$. In paper [4] 3D model assured that $\rho \propto T$ for T>100 K, which is below the Debye temperature.[4] To test the validity of the temperature dependence

of resistivity experiment were carried on which were in harmony with the theory.



Graph-2

Calculated temperature dependent resistivity due to twiston scattring. The upper Bold curve is calculated for one dimensional model for which $\rho\ \alpha T$. Including 3D (three dimensional) inter tubes effects in both the electrons and twiston degrees of freedom we obtained the lower curve which shows $\rho\ \alpha T$ in 3D sample relatively above low crossover temperature.[4]

The above discussion and references are made in order to accept the linear temperature dependence resistivity so that a quantum state of the electron with in CNT can be related with temperatue of CNT.In an attempt to do this, we simply introduce a very crucial thermo-electric dimensional quantity called temperature co-efficient of resistance below in the theoretical approach..Further introducing Joule's law of heating effect, we try to calculate heat evolved across. the conductor by simply substituting the quntized resistivity and temperature dependence quantum state

II. METHOD AND MODEL

For dealing with our problem we consider previous model of CNT with the one dimensional approach of carbon nano tubes in which electron is with in the well of infinite depth ,so that electron is completely free to to move with in the well .That means electron is confined from tranverse directions in the tube resuls in the quantized nature of the total energy[1].Hence one dimensional box model with single electron constitute the stating point for fruitful approach to find the quantized resistivity. As a methodology, we study the variation of resistivity with temperature of single wall one dimensional as well three .Our study is some extent empirical and some extent experimental.Empirical in the sense we adhere to previous data tabulated in table no 1 for study of variation of resistivity of different kinds of materials and observations referring to variation of resistivity with temperayure made in reference [4]. Honestly speaking, experimental cross verification was though not done, my approach was based on different experimental resuls already carried out such as , for instance see ref[4].With the cumulative approach in addition to well stablished result of my theoretical previous work, I try to develop and reproduce some classiacal results interms of quantum states.

III. theory

With the help of relation (v) we can find the quantum resistance of the carbon nano tube which is given by $R_n = \rho_n \frac{L}{A} = \frac{Anh}{2Le^2} \times \frac{L}{A}$

This value agrees with firs two values for n=1 and n=2 observed in ref. [4]

$$R_n = \frac{nh}{2e^2} \text{...............}(vi)$$

Let us now, bounce back to simply define $\propto$ (temperature co-efficient of resistance) Let, R_T, R_0 be the resistance at T^0c and $T_o{}^\circ c$ respectively then,

$$\propto = \frac{R_t - R_0}{R_0\left(T - T_0\right)} \text{................}(vii)$$

Relation (vi) makes it clear that R_n depends on the quantum state of the electron of CNT, that is $R_n \propto n$. But in carbon nano tubes as well as in other conductors the resistivity of the material depends on the number density of the electrons and average relaxation time which is given by the relation $\rho_n = \frac{m}{N_n e^2}\tau$ but more accurately for CNT $\rho_n = \frac{m}{2N_n e^2}\tau_n$ [3]. It follows that the resistivity of carbon nano tubes is inversely proportional to the average relaxation time of the free electron, since it is not a constant parameter and changes with change in temperature of the CNT. In fact, if the temperature of the CNT increases the amplitudes of the vibration of the lattice ions increase in addition to the increase of the speeds of free electrons. Consequently, free electrons collide more rapidly with vibrating lattice ions results in decrease of the relaxation time. Therefore, resistance of the tube increases as the temperature increases. Since, the resistance as per relation (vi), can only be changed when the quantum state 'n' is changed but change of temperature also changes the resistance R_n , hence resistivity. In other words, 'n' quantum state (n = 1, 2, 3,…) is a function of temperature. If 'R_t' is resistance at temperature $T^\circ c$ corresponding to the quantum state n, Similarly R_0 is the resistance at temperature $T_0{}^0c$ corresponding to quantum state n_0 . So, $R_t = \frac{nh}{2e^2}$ and $R_0 = \frac{n_0 h}{2e^2}$. So, relation (vii) can be put in terms of the quantum state. $\propto = \frac{\left(\frac{nh}{2e^2} - \frac{n_0 h}{2e^2}\right)}{\frac{n_0 h}{2e^2} \times \left(T - T_0\right)}$

Or, $\propto = \frac{\left(n - n_0\right)}{n_0\left(T - T_0\right)} \text{...........}(viii)$

$$\propto = \left(\frac{n}{n_0} - 1\right)\left(T - T_0\right)^{-1}$$

$$\Rightarrow \left(T - T_0\right)\propto = \frac{n}{n_0} - 1$$

$$\Rightarrow \frac{n}{n_0} = \left(T - T_0\right)\propto + 1$$

$$\Rightarrow n = \frac{1}{n_0}\left(T - T_0\right)\propto + \frac{1}{n_0} \text{...........}(ix)$$

$T_0 = 0^0$c then the above equation reduces to

$$n = \frac{1}{n_0}\left(T\alpha + 1\right) \text{.............}(x)$$

Transforming Celsius scale in to Kelvin scale has no impact on the value of quantum state as $\left(T - T_0\right)$ is only the difference in a given scale. Further, when a conductor having some resistance is connected to a source of E.m.f , a current sets up through the conductor and when the current flows through it, charge carrier (electrons) frequently collide with the other free electrons and

positive ions of the lattice as a result a part of their kinetic energy is converted in heat, thereby raising the temperature of the conductor. The same thing can be expected for a carbon nano tube if it is assumed to behave as a conductor.

Let us calculate the amount of power supplied in the resistance R_n which is in fact the work done by the source in deriving the current 'i' through the circuit containing CNT. If *dv* is the potential difference between the two ends of carbon nano tube while used in a circuit then we can write $dv = -\int_0^L \vec{E}.d\vec{l}$. The work done in moving a charge *dq* from one end to another end of the conductor $dw = dv.dq = v_{p.d}.dq$ since power consumed across the nano tube is equal to rate of work done by the source $dqP = \frac{dw}{dt} = v_{p.d}.\frac{dq}{dt}$

$$\Rightarrow P_{in} = V_{pd}.i \Rightarrow P_{in} = i.R_n.i \Rightarrow P_{in} = i^2 R_n$$

P_{in} is the instantaneous power consumed across the conductor. Therefore, total power absorbed in time't' is total work done by the source in that given time. If we assumed the whole of work done is converted in to heat energy in SI unit we can write $H = W = \int_0^t P_{in}.dt = \int_0^t i^2 R_n.dt ....(xi)$ but current through the nano tube $i = N_n A v_d.e..........(xii)$ where v_d is the drift velocity of the electron and *e* is the charge of the electron. Now, deriving the expression for drift velocity is time consuming and tedious, so we just refer to reference [3] for the expression of drift velocity, $v_d = \frac{2V_{pd}e}{AnhN_n}$ using the value of v_d in relation *(xii)* we have

$$i = N_n A.\frac{2V_{pd}e}{AnhN_n}e \Rightarrow i = 2V_{pd}\frac{e^2}{nh}..........(xiii)$$

$$i^2 R_n = \left(2V_{pd}\frac{e^2}{nh}\right)^2 \times \frac{nh}{2e^2} \Rightarrow i^2 R_n = 2V^2{}_{pd}\frac{e^2}{nh}........(xiv)$$

$$(xi) \Rightarrow H = \int_0^t 2V^2{}_{pd}\frac{e^2}{nh}dt$$

$$\Rightarrow H = 2V^2{}_{pd}\frac{e^2}{nh}\int_0^t dt..........(xv)$$

(Since we are assuming constant power supply i.e $V_{pd.} = \text{constant}$ and n is independent of time),

Mean power consumed $\overline{P} = \frac{H}{t} = 2V^2{}_{pd}\frac{e^2}{nh}.......(xvi)$

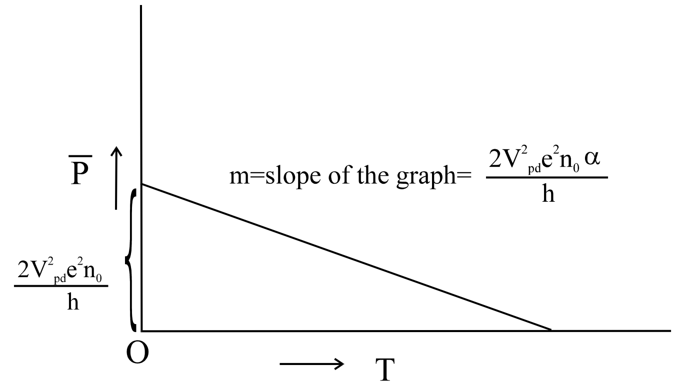
This is the expression for mean power consumed in CNT. Further

$$\overline{P} = \frac{H}{t} = 2V^2{}_{pd}\frac{e^2}{h(1+\alpha T)}n_0 = 2V^2{}_{pd}\frac{e^2}{h}n_0(1+\alpha T)^{-1}$$

$$\overline{P} = 2V^2{}_{pd}\frac{e^2}{h}n_0\left[1-\alpha T+(\alpha T)^2-........\right],$$

Since α is very small generally of the order 10^{-2} to10^{-4} see the table no. 1.

Therefore, $\overline{P} \approx 2V^2{}_{pd}\frac{e^2}{h}n_0(1-\alpha T).........(xvi)$



This is the approximate expression for power loss at temperature T, which decreases linearly with rise of temperature $\frac{2V^2{}_{pd}e^2n_0}{h} = k$ (say)

$$\overline{P} \approx k(1-\alpha T)$$

$$\frac{d\overline{P}}{dT} = -k\alpha............(xvii)$$

Which shows negative temperature gradient of power loss is equal to some constant times of the temperature co-efficient of the resistance. Since for no any value of temperature $\frac{d\overline{P}}{dT} = 0$ *i.e* $\frac{d\overline{P}}{dT} \neq 0$. Therefore power loss is not constant with respect to temperature which in fact means there is nothing like maximum or minimum power loss due to change of temperature i.e power loss goes on continuous decreasing with rise of temperature. Therefore, rate of power loss

$\frac{d\overline{P}}{dt} = \frac{d(k-k\alpha T)}{dt} \Rightarrow \frac{d\overline{P}}{dt} = 0$ ....(*xix*), in fact T – temperature and *t*-time are two independent variables as because when heat is developed across the CNT, with passage of time *t* then due to its good and excellent thermal conductivity, heat passes away and therefore there is no scope for rise of temperature across the CNT. Hence temperature T is supposed to be independent of time.

Now, instantaneous power consumed across the CNT.

$$(xv) \Rightarrow H = 2V^2{}_{pd}\frac{e^2}{nh}\int_0^t dt$$

$$(xv) \Rightarrow H = 2V^2{}_{pd}\frac{e^2}{nh}\frac{d}{dt}\int_0^t dt \Rightarrow \frac{dH}{dt} = 2V^2{}_{pd}\frac{e^2}{nh}\frac{d}{dt}\int_0^t dt$$

$$\Rightarrow P_{ins} = \frac{dH}{dt} = 2V^2{}_{pd}\frac{e^2}{nh}\frac{dt}{dt}$$

From

$$\Rightarrow P_{ins} = 2V^2{}_{pd}\frac{e^2}{nh}.......(xx)$$

relation (xvi) and (xx) $P_{ins} = \overline{P}$

The striking fact about CNT is that the instantaneous power is equal to mean power loss due to flow of current

$$\overline{P} = k(1+\alpha T)^{-1} = k\left[1-\alpha T+(\alpha T)^2-......\right]$$

If (αT) does not fall of as rapidly as we assumed for higher order terms still we can ignore $(\alpha T)^3$, $(\alpha T)^4$ ….. terms. Moreover, since instantaneous power and mean power is the same, so we term it as only power P

$$P \approx k\left[1-\alpha T+(\alpha T)^2-......\right]$$

$$\frac{dP}{dt}=-k\alpha+2k\alpha T$$

To check for maximum or minimum power loss at particular temperatures, Let us, put $\frac{dP}{dt}=0$

$\Rightarrow k\alpha(2T-1)=0 \;\; T=\frac{1}{2}=.5$ i.e at T=.5 degree celcius or Kelvin , we have either maximum or minimum power loss to be confirm, let us take the second derivative of the power P. i.e $\frac{d^2P}{dT^2}=2k\alpha$

Since k >0, α>0 for our case of conducting CNT, Therefore $\frac{d^2P}{dT^2}>0$

Power loss is minimum as the second derivative of power loss is positive *One important consequence of the above result is that when there is a temperature difference between the initial and final state approximately* $T-T_0=T=.5°c \; or \; .5° K$ as $T_0=0°c$ *,then obviously the power loss is minimum corresponding to the temperature.*

Heat loss over a period of relaxation time of electron τ_n

Now, relation (xv) implies

$$H=\int_0^{\tau_n}\frac{2V^2{}_{pd}e^2}{nh}dt$$

$$\Rightarrow H=\frac{2V^2{}_{pd}e^2}{nh}\tau_n \text{ as } V_{pd} \text{ is independet of time}$$

Now, substituting the value

$$\Rightarrow H=\frac{2V^2{}_{pd}e^2}{nh}\times\frac{2mL^2}{nh}$$

$$\Rightarrow H=\frac{4mV^2{}_{pd}L^2}{n^2h^2}........(xxi)$$

Then heat loss is found to be varying with the square of length of the CNT. Apart from this the heat lost over any period of time can be expressed in terms of relaxation time of the electron since any arbitrary time can be expressed in terms of relaxation time as 't'=C τ_n then heat lost over any period $H=\frac{2V^2{}_{pd}e^2}{nh}\times C\tau_n \Rightarrow H=\frac{4mCV^2{}_{pd}L^2}{n^2h^2}$ …..(xxii)

Thus heat lost for any period of time is C times to that of heat lost over a period of relaxation time, where C is the ratio of arbitrary time and relaxation time. Further, expanding the relation (xxi) we have,

$$\Rightarrow H=\frac{4mV^2{}_{pd}L^2n^2{}_0e^2}{h^2}\left[1-2\alpha T+3(\alpha T)^2+......\right]$$

So, mean power lost can be expressed as follows :

$$\Rightarrow \overline{P}=\frac{H}{\tau_n}=\frac{2mV^2{}_{pd}e^2L^2n^2{}_0(1+\alpha t)}{\tau_n n_0 h^2}\left[1-2\alpha t+3(\alpha t)^2+......\right]$$

$$\Rightarrow \overline{P}=\frac{2V^2{}_{pd}e^2n_0}{h}.\left[1-2\alpha T+3(\alpha T)^2+\alpha T-2(\alpha T)^2......\right]$$

$$\Rightarrow \overline{P}=\frac{2V^2{}_{pd}e^2n_0}{h}\left[1-\alpha T+(\alpha T)^2+......\right]$$

Table for Temperature Co-efficient of Resistance at 0^0 and 20^0c **0**

Table No-1

Material	α at 0^0c	α at 20^0c
Metal	$\times10^{-3}/°C$	$\times10^{-3}/°C$
Silver Copper Aluminum Gold Iron Platinum Tungsten	~3.8 ~3.9 ~3.9 ~3.4 ~5 ~3.92 ~4.5	~3.8 ~3.9 ~3.9 ~3.4 ~5 ~3.92 ~4.5
Material semiconductor	α at 0^0c	α at 20^0c
Carbon Germanium Silicon	-0.5×10^{-3} -48×10^{-3} -75×10^{-3}	-0.5×10^{-3} -48×10^{-3} -75×10^{-3}
Material Insulators	α at 0^0c	α at 20^0c
Wood Glass Mica	- - -	- - -
	$/°c$ = per degree celcius	(-) means data is not available

FINDININGS OF RESULTS

My finding although is pure mathematical, the physical aspect is supported by the experimental fact that the linear behav iour in respect of resistivity , quantum state of CNT with temperature is well established fact [4], there fore it is not surprising to see power loss too depends on temperature almost linearly but with negative slpoe .It is logical to argue that rise of temperature though enhances resistance in the CNT, it simultaneously reduces the current through the CNT as long as we are dealing with conducting CNT, since current in Joule's law appear i^2 ,whereas resistance with power unity, R, hence there is a fall of power loss due to increase of temperature.If the linear fall of continuous power loss is true then the remaining three result as mentioned earlier must be true.

discussion and remarks

With our semi classical approach, despite of its adequate simplicity, we have tried to simulate the observed temperature dependent resistivity which in turn establishes the linerar dependence of the quantum state of the electron in CNT under a few restrictions such as √A< <L and ∝ is a constant over a wide range of temperatures and conditions With the help of the linear relation of quantum state with the temperature, we have achieved three striking results in my research work. The first of all is the linear relationship of power loss with temperature with negative slope. The second one seems to be nicer that states, *Negative temperature gradient of power consumption is equal to some constant ($2V_{pd}^2\,e^2n_0/h$) times of the temperature coefficient of resistance of CNT. Last but not the least we have shown heat lost across the tube varies directly as the square of the length of the tube. However, exceptions may be there in the results due to the* Perturbation to the small extent of the presumptions and conditions of the physical aspects in internal structures in CNT. Finally in short, the exceptions to our result if any, can be attributed to the structural defects and internal inter actions arising out of electro electron

interactions, electron –ions interactions and interactions of the composite layers or ropes of the tubes et[2]and[4]c. It is .how ever logical to argue that linear electrical behavior of single wall one dimensional carbon nano tubes still holds good in ideal situation which can be a fruitful starting point of further research in the field of thermo electricity for critical appraisal and comparison of results with those of obtained in ordinary conductor to have optimum utility of CNT in applied aspects of life.

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